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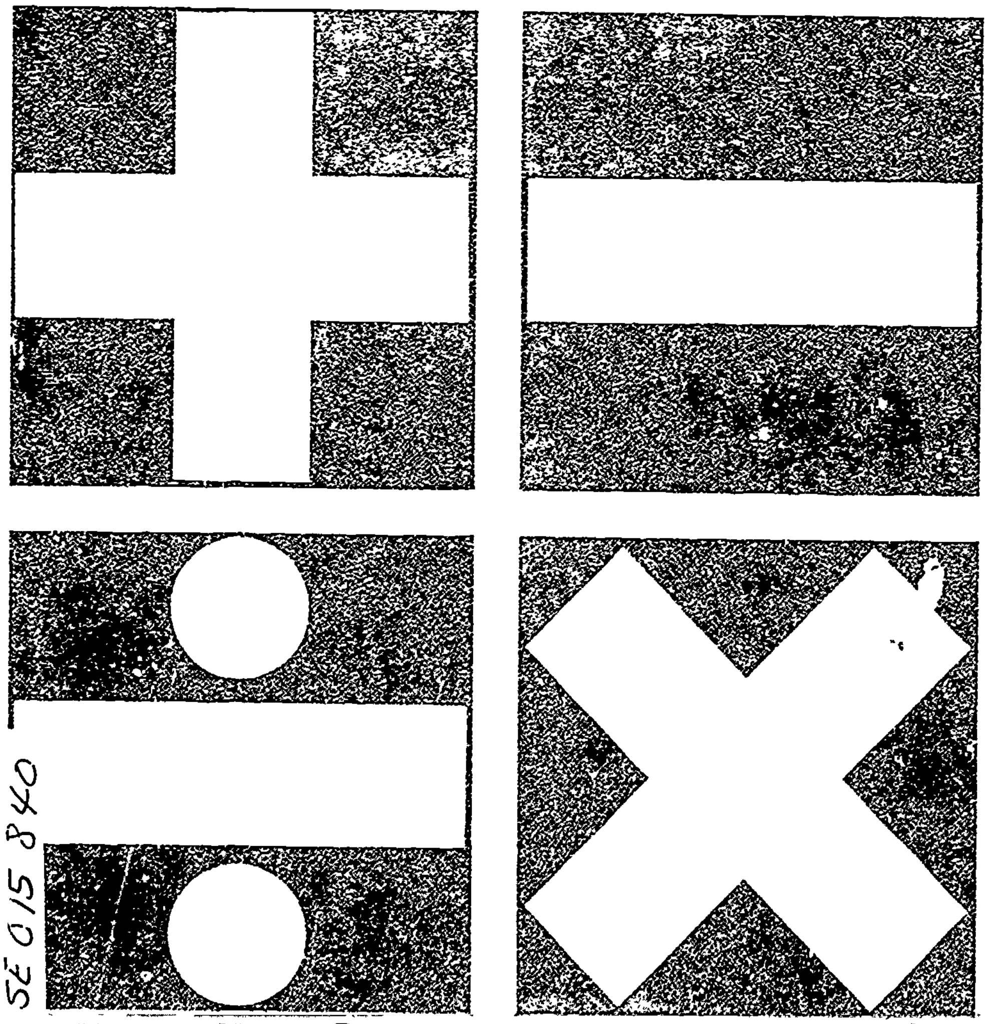
ABSTRACT

Written to provide some basic information for parents about the New York State Elementary School Mathematics Program, this pamphlet briefly discusses techniques of number operations on the whole numbers, number systems, sets, and geometric concepts. (DT)

ELEMENTARY SCHOOL

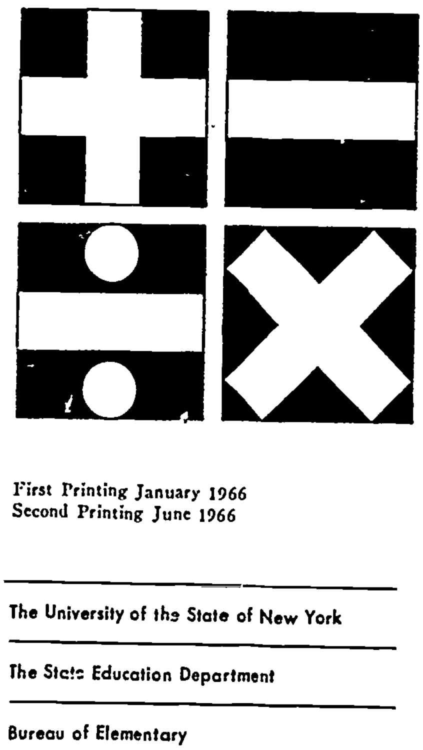
Mathematics

A Parents' Guide



A Parents' Guide

ELEMENTARY SCHOOL Mathematics



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Prefatory Note

Much of teaching has always been geared to the past—some to the present, and little to the future.

That the wisdom of the past be preserved, that today's purposes be adequately served, and that teaching be so guided as to prepare today's children to be adequate to the tasks of tomorrow, a new program in mathematics for the elementary grades has been prepared by the Bureau of Elementary Curriculum Development.

All parents share in this endless quest to make learning for children all that it can be.

January 1966

Walter Crewson

Associate Commissioner for Elementary, Secondary and Continuing Education

Contents

Foreword

Elementary School Mathematics, A Parents' Guide has been written to provide some basic information about the New York State Elementary School Mathematics Program. Authoritative sources of additional information have been included to aid parents who wish to delve further into the subject.

This new approach in mathematics has brought excitement to the learning of material that was previously taught too often in a rigid and tedious manner. Material is included in the program that stretches the creative and imaginative ability of every child. Mathematics is now becoming one of the favorite subjects of many elementary school children. The choice has been indicated on several nationwide surveys of children's preferences.

The Department has had the counsel and assistance of many individuals in the preparation of these materials. This manuscript was written by John Crensen, Supervisor of Mathematics Education at the Hicksville Public Schools. The material was prepared for publication by Dr. Robert A. Passy of the Bureau of Elementary Curriculum Development. Frank S. Hawthorne, Chief of the Bureau of Mathematics Education, and John J. Sullivan of the same bureau reviewed the manuscript. The Department is grateful to all those who assisted in the work.

Robert H. Johnstone
Chief,
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William E. Young
Director,
Curriculum Development Center

A Timetable of Change

> Two-thirds of skilled and semi-skilled job opportunities on the labor market today are beyond the reach of those who do not have an understanding of the basic principles of arithmetic, algebra, and geometry.

Mathematicians and teachers are working together to weld good teaching techniques to important mathematical ideas. They meet at university centers to share ideas and to plan. Then, they return to their schools to try out the material they develop, together with the children. Changes are made in the plan of teaching, based on what happens in the classroom. Answers are sought to such questions as:

> What important new material should be taught to children?
>
> What are the best ways of helping children learn mathematics?
>
> How can mathematics help the children now and in the future?

This type of work has been going on at many centers throughout our country. Many important findings have come from this effort.

The task of gathering these findings and appraising them was formally undertaken by the New York State Education Department in 1962. An ad hoc Advisory Committee on Mathematics met in Albany for this purpose. The committee was made up of representatives from the various new programs in the teaching of mathematics, outstanding mathematicians, and experienced teachers. Dr. Walter Crewson, Associate Commissioner of Education, chaired the meetings.

The recommendations of the ad hoc Advisory Committee on Mathematics were organized into an experimental course of studies. One hundred school districts in New York State participated in an

actual classroom test of the new instructional material. The evaluations made by the classroom teachers and the advice of the mathematics committee became the basis of the new mathematics program that was presented to New York State elementary schools by the New York State Education Department at the beginning of the 1964-65 school year. This new program of study represents a combined effort that includes 100 of our New York State elementary schools, many leading mathematicians, and the New York State Education Department. It takes account of the educational change that has taken place during recent years.

Pinpointing Important Ideas

Skill in computing (adding, subtracting, multiplying, and dividing) is an important goal in the new program. Children are guided to work with understanding and efficiency. This is done with the use of fundamental mathematical ideas.

Mathematics is a group of ideas related to each other. These ideas are consistent for algebra, advanced mathematics, and also elementary school mathematics. The child works with these ideas right at the beginning of his work in the subject. The new mathematics program systematically develops these principles throughout the total school experience of each child.

Guided Discovery

The new elementary mathematics program places great emphasis on *how* children are taught. Teachers are encouraged not to rely wholly upon *telling* as the chief or only method of instruction. Increased use of guided discovery will enable the learner to be more active in the process of learning. Instead of being told a rule or a method with which to arrive at a conclusion, children will be guided through activities from which meanings and understandings are developed.

Objects that can be handled and explored for their quantitive aspects are important for children to use before getting to their work with numerals. Wooden blocks, sticks, or discs may be used when the question is asked: Can you show me what this example means? Using objects to provide meaning to mathematical ideas is accorded a basic place in helping the child learn.

Children are told to: draw a picture of this numerical problem; show what this means with a drawing. The child will have opportunity

to refine his own ideas about quantity. He will have many activities with objects and illustrations. Work of this type also enables the teacher to know whether the child is capable of demonstrating his understanding of number.

The final step in this learning sequence is concerned with the use of numerals, which are symbols representing numbers. This work begins only when the child has an understanding of what the numerals mean. Our system of numeration uses 10 symbols: 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. Numerals may be used to tell us what is first (1st), second (2d), third (3d), and so on in a sequence of objects or ideas. Recipes and many of the products we buy use the order relationship to direct us in the task of putting things together in the right sequence. When numerals are used to refer to the order things are in, they are thought of as stating an *ordinal* relationship.

Numerals also transmit meaning of "how many." Two is one more than one, three is one more than two, and the number symbols are used as a way of comparing quantities. When we answer the question, "How many?" we are exploring the cardinal relationship that is described with the use of number symbols.

Individual Differences

Individual differences among children are carefully provided for in the new program of studies. Meaning is always developed at the onset of any topic in the course. Objects and pictures are used by the teacher whenever a child demonstrates a lack of understanding of the numerals or the computing process. Individual children will vary in the amount of time they need to work with objects for the purpose of gaining meaning. Teachers will return to the use of pictures and objects at each grade level for the purpose of relating mathematical ideas to materials the child can see and feel. Work in mathematics involves a great deal of work with objects and pictures, and is not limited to the use of number symbols.

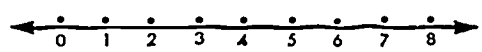
Techniques of Addition and Subtraction

Addition and subtraction are closely related computational tasks. Addition can be thought of as a "putting together" activity, while subtraction is a "taking away."

Using the Number Line

A number line is used as one of the ways to help the child see what is happening when addition or subtraction is taking place. It may take the following form.

A Number Line

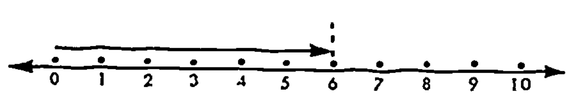


The arrowheads are used to show that the number line extends further than it has been drawn.

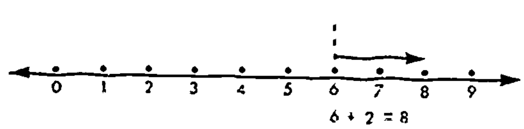
This example may be illustrated by counting to the right from the point marked 0. In certain cases, to subtract, the counting on the number line would be to the left.

Addition on a Number Line

$6 + 2 = ?$

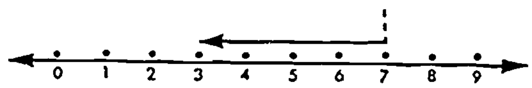


From 0 count 6 to the right. To add 2 to 6, count 2 from the stopping point (6) to the right, ending at 8.



Subtraction on a Number Line

$$7 - 4 = ?$$



Starting at 7, count 4 to the left for subtraction.

$$7 - 4 = 3$$

Working with the number line enables the child with knowledge of counting to add and subtract without having memorized the number facts. The child does not have to rely on memory. He has another way of reaching an answer.

As the child grows in number understanding, he is encouraged to add and subtract mentally. Answers should be quick, and counting to reach a conclusion is discouraged. Memorizing important combinations of numbers is necessary for efficient computing.

Using Mathematical Ideas

Basic mathematical ideas are used in the new program to enable the child to learn with more understanding. He learns:

$$3 + 2 = 5$$
$$2 + 3 = 5$$
and
$$3 + 2 = 2 + 3$$

The order in which the two numbers in the example are added does not change the sum. Sometimes the mathematical term, commutative, is used to refer to this property of addition. The child is guided to realize that adding 3 and 2 in either order will provide 5 as an answer. This commutative principle applies to the addition of any two of the numbers that the children work with in the elementary school. Knowledge of this idea aids the child in mastering the basic number facts for addition. If he knows $2 + 3 = 5$, then he also knows $3 + 2 = 5$. He cuts memory work in half.

Another important mathematical idea related to addition can be illustrated with the following example.

$$3 + 2 + 4 = ?$$

The way in which the addends are grouped does not affect the sum. Here, parentheses are used to signal that what is in the parentheses is done first.

$$(3 + 2) + 4 = ?$$
$$5 \quad + 4 = 9$$

or

$$3 + (2 + 4) = ?$$
$$3 \quad + 6 \quad = 9$$

This example illustrates a property that mathematicians call associativity. Children frequently make use of both associative and commutative properties to simplify computation. Finding the sum of

$$1 + 2 + 3 + 4 + 5 + 6 + 7 + 8 + 9 = ?$$

$$1 + 9 = 10$$
$$2 + 8 = 10$$
$$3 + 7 = 10$$
$$4 + 6 = 10$$
$$5 = 5$$

Or

$$10 + 10 + 10 + 10 + 5 = 45$$

has been done by grouping the addends. Children are given many opportunities to explore the possibilities of various number combinations.

Renaming

Various number combinations are used by the children as they become aware that a number has many names. For instance:

Other Number Names for 8

$0 + 8$	$16 \div 2$
$1 + 7$	4×2
$2 + 6$	$5\frac{1}{2} + 2\frac{1}{2}$
$3 + 5$	$1 + 3 + 4$

There are many more number names for eight. The process of expressing a number in various ways is sometimes called renaming. This is useful in adding:

$$312 + 429 + 86 + 47 = ?$$

Renaming

$$\begin{aligned} 312 &= 300 + 10 + 2 \\ 429 &= 400 + 20 + 9 \\ 86 &= 80 + 6 \\ 47 &= 40 + 7 \end{aligned}$$

Ones, tens, and hundreds are placed in separate columns. Work like this shows whether the child understands the meaning of each written symbol. The addition would then be done for each of the three columns. The sum for the example could be reached by then adding the three sums.

Left Hand Addition

Another way of finding a sum has been given the name of "left hand addition." It is not necessary to know how to "carry" with this method.

312	
429	
86	
+ 47	
700	Sum of the hundreds column
150	Sum of the tens column
24	Sum of the ones column
874	Total

Speed in computing is not essential at the beginning, when the goal is understanding. Eventually, the common way to reach a solution is introduced when the child demonstrates understanding of the process. Time has been devoted to building up the rationale of the addition technique.

Subtraction

Addition and subtraction are taught together because of their close mathematical relationship. Subtraction is sometimes considered the "undoing" of addition.

$$\begin{aligned} 5 &= 3 + 2 \\ 3 &= 5 - 2 \\ 2 &= 5 - 3 \end{aligned}$$

Children often work at learning addition and subtraction facts together.

Renaming is also used in helping the child to understand what "borrowing" means when subtraction is being done.

Renaming

$$\begin{array}{r} 42 \\ -\ 34 \\ \hline \end{array}$$

42 can be renamed as $40 + 2$
and $30 + 12$
34 can be renamed as $30 + 4$

The Example May Become

$$\begin{array}{r} 30 \\ -\ 30 \\ \hline \end{array} \qquad \begin{array}{r} 12 \\ -\ \ 4 \\ \hline \end{array} \qquad \begin{array}{l} (30 + 12 = 42) \\ (30 + \ \ 4 = 34) \end{array}$$

Another opportunity is given the child to demonstrate his knowledge of the different names a number may have, and to choose a suitable equivalent expression for the example being worked on. After using this method for a short time the child will do the problem in the usual manner:

$$\begin{array}{r} 42 \\ -\ 34 \\ \hline 8 \end{array} \quad \text{or} \quad 42 - 34 = 8$$

When working with subtraction, the idea of order (commutativity) that was operative when addition was undertaken is tested.
We have seen:

$$4 + 6 = 10$$
$$6 + 4 = 10$$
$$4 + 6 = 6 + 4$$

Looking at subtraction:

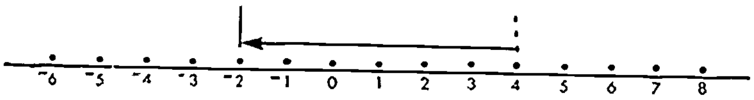
$$6 - 4 = 2$$
$$4 - 6 = -2$$
$$6 - 4 \text{ is not equal to } 4 - 6$$

The order principle does not apply to subtraction. (The expression, $4 - 6$, is equivalent to the negative number -2. Negative numbers may be compared to below zero readings on the thermometer. Work with such numbers is done in the upper elementary school grades.)

Other important mathematical principles are tested under the guidance of the teacher to see if they apply to subtraction. There is

a continual review of important ideas. Each new computational setting provides this opportunity.

We can use the number line to show that $4 - 6 = -2$



Multiplication and Division

Multiplication and division are also taught as closely related computational activities. Division may be thought of as the inverse operation (undoing) of multiplication. A child learns the number combinations in multiplication and division at the same time.

$$2 \times 3 = 6$$
$$3 \times 2 = 6$$
$$6 \div 3 = 2$$
$$6 \div 2 = 3$$

The phrase "factor $\times$ factor equals product" is used when we give names to the multiplication process. The parts of a division example would be labeled as a "product $\div$ factor $=$ factor." The same names are used to stress relatedness of a division to multiplication.

Factor	$\times$	factor	$=$	product
2	$\times$	3	$=$	6
2	$\times$	4	$=$	8
2	$\times$	5	$=$	10

Product	$\div$	factor	$=$	factor
6	$\div$	2	$=$	3
8	$\div$	2	$=$	4
10	$\div$	2	$=$	5

Commutativity with respect to multiplication and division is explored. The child will see:

$$2 \times 3 = 6$$
$$3 \times 2 = 6$$
$$2 \times 3 = 3 \times 2$$

The order in which the factors are arranged does not affect the product.

He will also note:

$$6 \div 3 = 2$$
$$3 \div 6 \text{ does not equal } 2 \text{, it equals } \tfrac{1}{2}.$$
$$6 \div 3 \text{ does not equal } 3 \div 6$$

Changing the order in a division example does affect the answer.

Multiplying

Multiplication may also be thought of as addition with certain kinds of problems.

$$2 \times 4 = 4 + 4$$
$$5 \times 3 = 3 + 3 + 3 + 3 + 3$$
$$3 \times 5 = 5 + 5 + 5$$

This is done to help the child develop understanding of what occurs when multiplication takes place. It is a beginning way of getting an answer without having to rely on memorized material. Later, the child is encouraged to commit the number facts for muitiplication to memory. Memorization is undertaken after the child has demonstrated an understanding of how the answers were obtained.

Renaming is also used to develop meaning and understanding of multiplication.

Renaming

$$\begin{array}{rrrl}
24 = & & 20 + 4 & \\
\underline{\times\ 15} = & & \underline{\times\ 10 + 5} & \\
 & & 100 & (\ 5 \times 20) \\
 & & 20 & (\ 5 \times 4\ ) \\
 & & 200 & (10 \times 20) \\
 & & \underline{40} & (10 \times 4\ ) \\
 & & 360 &
\end{array}$$

This way of computing stresses that the 1 in 15 really means 10. Multiplication is actually done with the 10. Also the 2 in 25 is actually referring to 20. After this introductory method of multiplication the children will use the usual method.

$$\begin{array}{r}
24 \\
\underline{\times\ 15} \\
120 \\
\underline{24\ } \\
360
\end{array}$$

Actually a zero was omitted when we multiplied 24 by 10. The more efficient way of computing is used after meaning has been developed. Starting with the advanced method would force the child to memorize a method without developing understanding. A new mathematics program carefully explores in intermediate steps what is actually being computed.

Division

Addition can be used to arrive at an answer in division. Multiplication is also first explored with the use of addition. This dependency of the various types of computational activities upon addition has led some mathematicians to state that addition is a basic arithmetical task, and subtraction, multiplication, and division are derivatives of the addition process.

$$438 \div 42 = ?$$

How many 42's will be necessary to have a sum equal 438?

$$\begin{array}{r} 42 + 42 + 42 = 126 \\ 42 + 42 + 42 = 126 \\ 42 + 42 + 42 = \underline{126} \\ 378 \end{array}$$

$$\begin{array}{r} 378 \\ \underline{+\ \ 42} \\ 420 \\ \underline{+\ \ 18} \\ 438 \end{array}$$

We have added ten 42's, and then added 18 to the total in order to reach 438.

$438 \div 42 = 10$ with 18 remainder

We are also able, with many problems, to find an answer to a division example with the use of subtraction. This involves a lengthy way of recording our computation on paper (mathematical term: *algorism*), but a conclusion is reached in a way that helps the child understand what is actually being done when you divide.

For example:

$$359 \div 72 = ?$$

We subtract 72 from 359

$$\begin{array}{r l}
359 & \\
-\ 72 & \\
\hline
287 & \\
-\ 72 & \text{And from each} \\
\hline
215 & \text{answer successively} \\
-\ 72 & \\
\hline
143 & \\
-\ 72 & \\
\hline
71 & \text{Until an answer is} \\
 & \text{less than } 72
\end{array}$$

$$359 \div 72 = 4 \text{ and } 71 \text{ remainder}$$

The children recognize that these are lengthy ways to seek an answer, and are thereby motivated to adopt the common method of division. They are urged to explore ways in which the job can be done quicker. Past learning is reviewed in order to find another way. Another method, though not the quickest, might be:

$$579 \div 28 = ?$$

$$\begin{array}{r l l}
 & \underline{10 + 10 \quad \text{or } 20} & \\
28 &)\,579 & \\
 & \underline{280} & (28 \times 10) \\
 & 299 & \\
 & \underline{280} & \\
 & \ \ 19 &
\end{array}$$

$$579 = 28 \times 20 + 19$$

The answer was reached with multiplication being done with tens.

$$579 \div 28 = 20\,\frac{19}{28}$$

The problem of where to put the first digit of the answer was avoided. No need was evident to "move over" and "bring down" because the multiplication was done in the way that was previously taught. Though tens were used in this example, any convenient number could have been chosen. A child of ancient Greece would probably have used 2 because the Greeks worked a great deal with multiples of two.

Various ways of dividing have thus been shown to the child. Understanding of what is being done when you divide has been carefully developed. Instead of there being just one way to reach a conclusion, the child has explored several of the many ways. The standard way

of computing and recording computations on paper (algorism) is finally taught as a fast and efficient computational technique.

$$
\begin{array}{rl}
 & \quad\ \ \ \ 17 \\
 & \quad 20\ \overline{23} \\
23 & \overline{)\ 477\quad\ \ \ } \\
 & \ \ \ \underline{46} \\
 & \ \ \ 17
\end{array}
$$

In the past, children often had difficulty with this method. Building up to it with simpler ways of computing an answer will assist them in using alternate ways to solve the example.

Number Systems

Children involved in a modern program delight in confronting their parents with riddles such as "When does $4 + 4 = 13$?" When such a question occurs, you know that your child is exploring number systems other than our conventional decimal system.

Ten basic symbols are utilized in our system: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9. These digits, as we group our quantities in sets of ten, enable us to name many numbers. Consider the following number of dots:

. . . .

There are thirteen. One group of ten dots and another of three. Thus, in the numeral 13, 1 represents ten while the 3 means three. In another form:

$$13 = 10 + 3$$

The position of a digit in a numeral is very important. Each successive place to the left represents a quantity ten times that of the preceding place. This is shown by the following table.

Thousands	Hundreds	Tens	Ones
$10 \times 10 \times 10$	10×10	10×1	1
1000	100	10	1

We are now able to rename 324 on the basis of the ideas developed in the preceding table.

Renaming 324

$$324 = 300 + 20 + 4$$

$$324 = (3 \times 100) + (2 \times 10) + 4$$

Renaming activities are undertaken in order to see whether the child understands our number system.

Other number systems are examined in the later grades after the child has developed full understanding of our decimal number system. Another system that is grouped in fives instead of tens might be one looked at. Here five basic symbols may be chosen. The familiar symbols: 0, 1, 2, 3, and 4 could be used. Our dots indicating 13 could be grouped as follows:

. . . .

Instead of grouping by tens, it has been done by fives. We have no symbol available to indicate five. The position of an available numeral could be changed to indicate five, just as we used 1 in another position to signify ten. Each successive place to the left in a numeral would signify a quantity five times that of the place to its right. Remember, we are thinking in terms of fives in the following table.

One-hundred twenty-fives	Twenty-fives	Fives	Ones
$5 \times 5 \times 5$	5×5	5×1	1
1000_{five}	100_{five}	10_{five}	1_{five}

The subscript "five" is used to indicate we are working in a system of numeration whose base is five, even though the symbols are the familiar ones—1, 2, 3, 4, 0. The number of dots is symbolized by the numeral 23_{five} (2 fives and 3 ones) in this system of numeration. Familiar ideas relevant to our number system are reviewed in a new setting to emphasize their universal applicability. Counting in a number system based on five is compared to our decimal number system in the following table.

Counting

Base Ten	Base Five	
1	1	
2	2	
3	3	
4	4	
5	10	(read one-zero, not ten)
6	11	(one-one)
7	12	(one-two)
8	13	(one-three)
9	14	(one-four)
10	20	(two-zero)

This type of work in the elementary school mathematics program helps the child review important material in a different setting.

Sets

Another "new" idea being introduced to children is that of *set*. It is included in the course of study for the kindergarten child, and is so all encompassing that it is considered a unifying idea in arithmetic, algebra, and geometry.

A set is well-defined if it is always possible to determine whether an object does or does not belong to the set. The set of presidents of the United States is well-defined. The set of women whose hair is naturally blond is not well-defined.

The primary school child, already familiar with the word set, uses it to describe any collection of objects—set of blocks, set of chairs, set of dishes, or set of bird pictures. In manipulating these sets of objects, the child is guided to a discovery of certain ideas related to arithmetic.

The most important concepts for the primary school child are those related to number. By matching sets on a one-to-one basis, such as having one chair for each child, one cookie for each paper plate, the understanding of quantity and the need for numerals to signify are developed. The development of arithmetical awareness begins with actual situations before exploring abstract ideas.

When an object is in a set (group or collection), a pupil would say that it is a member or element of the set. Braces are used to enclose the members of a set, and commas are used to separate the members of a set when we are describing a set in written form. For example:

The set of common vowels $= \{a, e, i, o, u\}$

The set of the first four counting numbers $= \{1, 2, 3, 4\}$

Sets are examined and compared for likenesses and differences. The set of common vowels contains five members, while the specific set of counting numbers has four members. Thus, the quantitive aspect of various sets is determined. Sets may be combined to form new sets, or separated into smaller sets than the original. These activities urge the pupil to seek out and think through responses that are not necessarily numerical in nature. Patterns for logically thinking about a problem are used with this method.

Geometry

What has happened to the study of geometry at the elementary school level? Students, once concerned with the endless memorization of formulas and units of measurements, are now eagerly engaged in the exploration of geometry. They are building a foundation that will serve them not only for future study in the subject but also for the practical applications to the physical world.

The approach is explorative with the prime concern, geometrical space and location in space. Pupils can, by their own observations of their physical environment, discover and develop the important ideas of geometry. To describe their observations and express these ideas, a basic vocabulary is needed. This vocabulary includes the geometric terms: *point, space, plane, curve, line, segment,* and *ray*.

> *Point* refers to a precise location. It has no dimensions (width, length, or depth). We indicate this location by a penciled dot; but, if the dot is erased, the location itself still remains.
>
> *Space* is the set of all points, or locations. The entire universe is dense with these points. These points, however, are fixed and do not move. As an object moves, it constantly occupies different sets of fixed points.

Points, represented by dots, are labeled with capital letters such as A • or B •. The shortest distance between these two points A •———• B is called a *line segment*, and labeled $\overline{AB}$. The line segment is a set of points and as such is considered a subset of geometric space.

Imagine a line segment that doesn't stop at either endpoint, but goes on endlessly in both directions. This is referred to as a *line* A ←———→ B, and labeled $\overleftrightarrow{AB}$. If I were concerned with only an end point and all points on the line extending in one direction from that point, I would have a *ray*, A •———•——→ B, and label it $\overrightarrow{AB}$. Two rays with a common endpoint make an *angle*.

Three points that are not on the same line determine a plane. Planes may be represented by such models as: desk tops, table tops, and the surfaces of floors. A plane may be thought of as a flat surface with unending length and width. It does not have thickness.

Other points between two points also arouse the curiosity of children. How many such possible paths do we have connecting $\overset{A}{\bullet}$ and $\overset{B}{\bullet}$?

We have an infinite number of such curves. Those illustrated above are *simple plane curves* because they do not intersect themselves or pass through the same point more than once.

A simple plane closed curve is one that starts at a point and returns to that point without crossing itself. For example:

Again, you have endless possibilities.

Those simple plane closed curves made up of three or more line segments have a special name—*polygons.* A three sided one is a *triangle;* four sided, a *quadrilateral;* five sided, a *pentagon;* six sided, a *hexagon;* and so on.

A simple closed curve separates the plane into three sets of points: the curve itself; the points enclosed by the curve or interior; and the set of points outside of the simple closed curve.

The extensions into space geometry, or three dimensions, follow the same logical, intuitive development as the two dimensional, or plane geometry. Through this approach, the child develops an understanding and appreciation of geometry. This knowledge becomes a foundation for future study.

In Conclusion

Children participating in the new elementary mathematics program are helped toward greater understanding of important mathematical ideas. Computational activities are developed in a manner that provides many ways of finding answers. Addition, subtraction, multiplication, and division take on a flexibility that is based on knowledge of the mathematical underpinnings of such activities. In the new program of studies, children are directed towards strengthened understanding and skill in computing.

Important new material is included in the program to provide for the anticipated demands of the future. The new content is always related to actual school settings. Newer ideas are incorporated into the school program as soon as they have proven workable in actual classrooms under test conditions.

A list of publications on *mathematics for parents* has been included in this publication. If one desires to learn more about mathematics, he should consult this listing. Your local school administrators are another source of information about the elementary school mathematics program.

Additional Sources of Information

Adler, Irving. *A new look at arithmetic.* Day. 1964

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Goggin, Robert L. *Modern math for parents.* Riverside, Calif. P. O. Box 4127. 1964

Heimer, Ralph T., and Newman, Mirian S. *The new mathematics for parents.* Holt. 1965

Huber, Alice, and Woods, Eileen. *Explaining new mathematics.* Kenworthy Educational Service. 1964

Rosenthal, Evelyn B. *Understanding the new mathematics.* Fawcett. 1965

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This booklet is published primarily for use in the schools of New York State, and free copies are available to New York School personnel when ordered through a school administrator from the Publications Distribution Unit, State Education Building, Albany, New York 12224.